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LETTER TO THE EDITOR

Towards gravitationally assisted negative refraction of light by vacuum

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Abstract
Propagation of electromagnetic plane waves in some directions in gravitationally affected vacuum over limited ranges of spacetime can be such that the phase velocity vector casts a negative projection on the time-averaged Poynting vector. This conclusion suggests, *inter alia*, gravitationally assisted negative refraction by vacuum.

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1. Introduction

The discovery of (purportedly) isotropic, homogeneous, dielectric-magnetic materials that bend electromagnetic rays the 'wrong way' [1] created quite a stir in 2001 [2], with claims and counterclaims flying all around [3, 4]. The situation has recently been settled, with unequivocal demonstrations by several independent groups [5–7]. See [8] for a comprehensive review. A range of exotic and potentially useful phenomena—such as negative refraction, negative Doppler shift and inverse Čerenkov radiation—have been predicted for materials of this type, wherein the phase velocity is directed in opposition to the energy velocity as quantified through the time-averaged Poynting vector. These materials have several names, including left-handed materials, negative-index materials and negative-phase-velocity (NPV) materials. We prefer the last term [8].

Subsequently, the possibility of NPV propagation of light and other electromagnetic waves was established in a variety of anisotropic materials [9–11]. In these materials, NPV propagation is indicated by the projection of the phase velocity on the time-averaged Poynting vector being negative.

Even more interestingly, materials that do not permit the observation of NPV propagation by observers in a relatively stationary (i.e., co-moving) inertial reference frame have been shown, after the invocation of the postulates of special theory of relativity (STR), to allow observation of NPV propagation in other inertial frames [12]. That permits one to envisage STR negative refraction being exploited in astronomical scenarios such as, for example, in the remote sensing of planetary and asteroidal surfaces from space stations. Quite possibly, space telemetry technologies will be the first to reap the benefits of STR negative refraction. Application to remotely guided, extraterrestrial mining and manufacturing industries can also be envisioned. Furthermore, many unusual astronomical phenomena would be discovered and/or explained via STR negative refraction to interpret data collected via telescopes [13].

As is well known, vacuum (i.e., matter-free space) appears the same to all inertial observers [14]. Therefore, as a co-moving observer cannot deduce the occurrence of NPV propagation in vacuum, neither can any observer moving with a constant velocity. This could lead one to believe that NPV propagation is impossible in huge expanses of interstellar space. However, gravitational fields from nearby massive objects will certainly distort electromagnetic propagation, which is a principal consequence of the general theory of relativity and is indeed used nowadays in GPS systems. Our objective here is to establish that gravitationally affected vacuum can support NPV propagation, at least in spacetime manifolds of limited extent.

2. Theory

A gravitational field curves spacetime, whose effect is captured through a metric $g_{\alpha\beta}$.[4] Electromagnetic propagation in gravitationally affected vacuum may be described in terms of propagation in an instantaneously responding medium in flat spacetime [15, 16], at least in spacetime manifolds of limited extent. That is, the *nonuniform* metric $g_{\alpha\beta}$ can be locally approximated by the *uniform* metric $\tilde{g}_{\alpha\beta}$ [17]. On assuming the convention $\tilde{g}_{\alpha\beta} = (+, -, -, -)$, the constitutive relations of vacuum in the equivalent flat spacetime are expressed in Gaussian units as [16]

$$D_\ell = \epsilon_{\ell m} E_m + \epsilon_{\ell mn} \tilde{g}_m H_n, \tag{1}$$

$$B_\ell = \mu_{\ell m} H_m - \epsilon_{\ell mn} \tilde{g}_m E_n, \tag{2}$$

where $\epsilon_{\ell mn}$ is the Levi-Civita tensor, and

$$\epsilon_{\ell m} = \mu_{\ell m} = -(-\tilde{g})^{1/2} \frac{\tilde{g}^{\ell m}}{\tilde{g}_{00}}, \tag{3}$$

$$\tilde{g}_\ell = \frac{\tilde{g}_{0\ell}}{\tilde{g}_{00}}, \tag{4}$$

with $\tilde{g} = \det[\tilde{g}_{\alpha\beta}]$. We note that the metric $\tilde{g}_{\alpha\beta}$ is real symmetric [18].

The constitutive relations (1) and (2) can be expressed in 3-vector form as

$$\underline{D} = \epsilon_0 \underline{\underline{\gamma}} \cdot \underline{E} - \frac{1}{c_0} \underline{\Gamma} \times \underline{H}, \tag{5}$$

$$\underline{B} = \mu_0 \underline{\underline{\gamma}} \cdot \underline{H} + \frac{1}{c_0} \underline{\Gamma} \times \underline{E}, \tag{6}$$

wherein SI units are implemented. The scalar constants ϵ_0 and μ_0 denote the permittivity and permeability of vacuum in the absence of a gravitational field, respectively, and $c_0 = \sqrt{1/\epsilon_0 \mu_0}$.

[4] Roman indices take the values 1, 2 and 3, while Greek indices take the values 0, 1, 2 and 3.

Our coordinate system is chosen such that the second-rank Cartesian tensor $\underline{\underline{\gamma}}$ is diagonal; i.e., $\underline{\underline{\gamma}} = \mathrm{diag}(\gamma_x, \gamma_y, \gamma_z)$. In the gravitational field of a mass rotating with angular momentum $\underline{J}$, the gyrotropic vector $\underline{\Gamma}$ is proportional to $\underline{R} \times \underline{J}$, where $\underline{R}$ is the radial vector from the centre of mass to the point of observation. By considering a small region of space at a sufficiently remote location from the centre of mass, we take $\underline{\Gamma}$ to be independent of $\underline{R}$.

We seek planewave solutions

$$\underline{E} = \mathrm{Re}\{\underline{E}_0 \exp[\mathrm{i}(\underline{k} \cdot \underline{r} - \omega t)]\}, \tag{7}$$

$$\underline{H} = \mathrm{Re}\{\underline{H}_0 \exp[\mathrm{i}(\underline{k} \cdot \underline{r} - \omega t)]\}, \tag{8}$$

to the source-free Maxwell curl postulates

$$\nabla \times \underline{E} + \frac{\partial}{\partial t}\underline{B} = \underline{0}, \tag{9}$$

$$\nabla \times \underline{H} - \frac{\partial}{\partial t}\underline{D} = \underline{0}. \tag{10}$$

Here $\underline{k}$ is the wave vector, $\underline{r}$ is the position vector, ω is the angular frequency and t denotes the time; whereas $\underline{E}_0$ and $\underline{H}_0$ are complex-valued amplitudes.

An eigenvector equation for $\underline{E}_0$ is developed as follows. By combining (5)–(8) with the Maxwell curl postulates, we derive

$$\underline{p} \times \underline{E}_0 = \omega\mu_0 \underline{\underline{\gamma}} \cdot \underline{H}_0, \tag{11}$$

$$\underline{p} \times \underline{H}_0 = -\omega\epsilon_0 \underline{\underline{\gamma}} \cdot \underline{E}_0, \tag{12}$$

in terms of

$$\underline{p} = \underline{k} - \omega\underline{\Gamma}. \tag{13}$$

The use of (11) to eliminate $\underline{H}_0$ from (12) provides, after some simplification,

$$\underline{\underline{W}} \cdot \underline{E}_0 = \underline{0}, \tag{14}$$

where

$$\underline{\underline{W}} = \left(k_0^2 \det[\underline{\underline{\gamma}}] - \underline{p} \cdot \underline{\underline{\gamma}} \cdot \underline{p}\right)\underline{\underline{I}} + \underline{p}\,\underline{p} \cdot \underline{\underline{\gamma}}, \tag{15}$$

and the notation $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ has been introduced. A dispersion relation thus emerges from (14) as

$$\det[\underline{\underline{W}}] = 0, \tag{16}$$

which may be expressed in the form

$$k_0^2 \det[\underline{\underline{\gamma}}]\left(k_0^2 \det[\underline{\underline{\gamma}}] - \underline{p} \cdot \underline{\underline{\gamma}} \cdot \underline{p}\right)^2 = 0. \tag{17}$$

Hence, we conclude that planewave solutions satisfy the condition

$$\underline{p} \cdot \underline{\underline{\gamma}} \cdot \underline{p} = k_0^2 \det[\underline{\underline{\gamma}}]. \tag{18}$$

Let us consider eigenvector solutions to (14). Substitution of (18) into (14) provides

$$\underline{p}\,\underline{p} \cdot \underline{\underline{\gamma}} \cdot \underline{E}_0 = \underline{0}; \tag{19}$$

thereby, all eigenvector solutions are necessarily orthogonal to $\underline{p} \cdot \underline{\underline{\gamma}}$. To proceed further, let us—without any loss of generality—choose the wave vector $\underline{k}$ to lie along the z axis, and the vector $\underline{\Gamma}$ to lie in the xz plane; i.e.,

$$\underline{k} = k\underline{\hat{u}}_z, \tag{20}$$

$$\underline{\Gamma} = \Gamma(\underline{\hat{u}}_x \sin\theta + \underline{\hat{u}}_z \cos\theta), \tag{21}$$

where $\underline{\hat{u}}_x$, $\underline{\hat{u}}_y$ and $\underline{\hat{u}}_z$ are the Cartesian unit vectors in the equivalent flat spacetime. Since

$$\underline{p} \cdot \underline{\underline{\gamma}} = -\omega\Gamma\gamma_x \sin\theta \underline{\hat{u}}_x + (k - \omega\Gamma\cos\theta)\gamma_z \underline{\hat{u}}_z, \tag{22}$$

it is clear that two linearly independent eigenvectors satisfying (18) may be stated as

$$\underline{e}_1 = \underline{\hat{u}}_y, \tag{23}$$

$$\underline{e}_2 = \underline{\hat{u}}_y \times (\underline{p} \cdot \underline{\underline{\gamma}}) \tag{24}$$

$$= (k - \omega\Gamma\cos\theta)\gamma_z \underline{\hat{u}}_x + \omega\Gamma\gamma_x \sin\theta \underline{\hat{u}}_z. \tag{25}$$

After assuming that $\underline{\underline{\gamma}}$ is invertible, we deduce the corresponding magnetic field eigenvectors from (11) as

$$\underline{h}_1 = \frac{1}{\omega\mu_0} \underline{\underline{\gamma}}^{-1} \cdot [(\omega\Gamma\cos\theta - k)\underline{\hat{u}}_x - \omega\Gamma\sin\theta \underline{\hat{u}}_z], \tag{26}$$

$$\underline{h}_2 = \frac{1}{\omega\mu_0} \underline{\underline{\gamma}}^{-1} \cdot [(k - \omega\Gamma\cos\theta)^2 \gamma_z + (\omega\Gamma\sin\theta)^2 \gamma_x] \underline{\hat{u}}_y. \tag{27}$$

Hence, the general solution is given by

$$\underline{E}_0 = C_1 \underline{\hat{u}}_y + C_2[(k - \omega\Gamma\cos\theta)\gamma_z \underline{\hat{u}}_x + \omega\Gamma\gamma_x \sin\theta \underline{\hat{u}}_z], \tag{28}$$

$$\underline{H}_0 = \frac{1}{\omega\mu_0} \underline{\underline{\gamma}}^{-1} \cdot \{C_1[(\omega\Gamma\cos\theta - k)\underline{\hat{u}}_x - \omega\Gamma\sin\theta \underline{\hat{u}}_z] + C_2[(k - \omega\Gamma\cos\theta)^2 \gamma_z + (\omega\Gamma\sin\theta)^2 \gamma_x] \underline{\hat{u}}_y\}, \tag{29}$$

wherein C_1 and C_2 are arbitrary constants.

The wave numbers arise from the dispersion relation (17) as follows. Substituting (13) into (18), we obtain the k-quadratic expression

$$k^2\gamma_z - 2k\gamma_z\omega\Gamma\cos\theta + \omega^2\Gamma^2(\gamma_x \sin^2\theta + \gamma_z \cos^2\theta) - k_0^2 \det[\underline{\underline{\gamma}}] = 0, \tag{30}$$

since $\underline{k} \cdot \underline{\Gamma} = k\Gamma\cos\theta$. The two k-roots of (30) are

$$k^+ = \omega \left(\Gamma\cos\theta + \sqrt{\epsilon_0\mu_0\gamma_x\gamma_y - \frac{\gamma_x}{\gamma_z}\Gamma^2 \sin^2\theta} \right), \tag{31}$$

$$k^- = \omega \left(\Gamma\cos\theta - \sqrt{\epsilon_0\mu_0\gamma_x\gamma_y - \frac{\gamma_x}{\gamma_z}\Gamma^2 \sin^2\theta} \right). \tag{32}$$

Finally, let us consider the time-averaged Poynting vector given by

$$\underline{P} = \tfrac{1}{2} \operatorname{Re}\{\underline{E}_0 \times \underline{H}_0^*\}. \tag{33}$$

After utilizing the general solution (28) and (29), the component of the Poynting vector aligned with the $\underline{\hat{u}}_z$ axis is obtained as

$$\underline{\hat{u}}_z \cdot \underline{P} = \frac{1}{2\omega\mu_0\gamma_z} (k - \omega\Gamma\cos\theta)(|C_1|^2 + |C_2|^2 \gamma_z \omega^2 \det[\underline{\underline{\gamma}}]). \tag{34}$$

The energy density flow in the direction of the wave vector $\underline{k}^+$, corresponding to the root k^+ given in (31), is thus

$$\underline{k}^+ \cdot \underline{P} = \frac{1}{2\mu_0\gamma_z} \left[\Gamma\cos\theta \sqrt{\epsilon_0\mu_0\gamma_x\gamma_y - \frac{\gamma_x}{\gamma_z}\Gamma^2 \sin^2\theta} + \left(\epsilon_0\mu_0\gamma_x\gamma_y - \frac{\gamma_x}{\gamma_z}\Gamma^2 \sin^2\theta \right) \right] \times (|C_1|^2 + |C_2|^2 \gamma_z \omega^2 \det[\underline{\underline{\gamma}}]). \tag{35}$$

Let us note that the inequality

$$\epsilon_0\mu_0\gamma_x\gamma_y - \frac{\gamma_x}{\gamma_z}\Gamma^2\sin^2\theta \geqslant 0 \tag{36}$$

must be fulfilled in order for k to be real-valued. Therefore, the defining inequality for NPV propagation, namely

$$\underline{k}^+ \cdot \underline{P} < 0, \tag{37}$$

is satisfied provided that

$$-\Gamma\cos\theta < \sqrt{\epsilon_0\mu_0\gamma_x\gamma_y - \frac{\gamma_x}{\gamma_z}\Gamma^2\sin^2\theta} \tag{38}$$

holds. Analogously, we find that NPV propagation is signalled for the k^- wave number by the condition

$$\Gamma\cos\theta < \sqrt{\epsilon_0\mu_0\gamma_x\gamma_y - \frac{\gamma_x}{\gamma_z}\Gamma^2\sin^2\theta}. \tag{39}$$

In deriving (38) and (39), we used the fact that $\gamma_{x,y,z} < 0$ by virtue of the signature of $\tilde{g}_{\alpha\beta}$.

We note that the NPV conditions (38) and (39) are independent of frequency.

3. Concluding remarks

The inequalities (38) and (39) can be satisfied for specific ranges of the angle θ, for given $\underline{\underline{\gamma}}$ and Γ. Thus, we have shown that NPV propagation in some directions is possible in gravitationally affected vacuum over limited ranges of spacetime. The possible existence of gravitational fields which can deliver $\underline{\underline{\gamma}}$ and Γ necessary for the satisfaction of (38) and/or (39) is a matter for astrophysicists to discuss.

We are content here to state that, just as scientific and technological applications of STR negative refraction (by materials) can be envisaged [12, 13], similar and different consequences of gravitationally assisted negative refraction by vacuum are possible. In particular, designers of channels for space communication shall also have to account for the possibility of negative refraction due to massive objects between the two ends of every channel.

Furthermore, current ideas on the distribution of mass in the as-observed universe may require significant revision, since electromagnetic signals from distant objects may be deflected in a manner which has not hitherto been accounted for. Thus, our work has implications for gravitational lenses [19]. Gravitational lensing involves nonuniform metrics, and the distribution of matter in the universe has been constantly changing. While the spatiotemporally *local* evolution of the universe may be deduced adequately from electromagnetic signals received by our telescopes, reasonably accurate deductions about the spatiotemporally *global* evolution of the universe from similar measurements may be particularly difficult to make—owing to gravitationally assisted negative refraction.

Finally, our work suggests that research on the consequences of gravitationally assisted negative refraction by materials should now be undertaken.

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